Pre-Calculus Summer Review

Welcome to Pre-Calculus!

While we certainly hope you have a good summer filled with fun and enriching activities, as well as plenty of time relaxing with family and friends, we also want to ensure everyone is entering Pre-Calculus on equal footing, and with a solid foundation that will help them start the school year on a positive note. For that reason, we have compiled a list of topics we have identified as the *most important* content from prior years. **Over the summer, your job is to make sure you are comfortable with these skills and topics <u>without the use of a calculator.</u>**

What is my assignment?

The work you do over the summer will be different for everyone. As you look at the topics listed, if you feel comfortable and confident with all the material, you may just choose to do a few practice problems from each topic to brush up at the end of August. If the topics listed seem unfamiliar, or you cannot even remember learning them in previous years, then you will have more practice to do over the summer to ensure you are ready for Pre-Calculus. In short – do as much or as little practice as needed (**and if you need or want even more than what is included here, feel free to reach out to your teacher**!), but please make sure you are comfortable with the material by the time school starts in September.

Why do I have to do this?

In math, just like in foreign languages, it is essential to review past material and keep it fresh in your brain, otherwise, it gets forgotten. Past material is also the building blocks of the new material we'll be learning! This means that by the time you get to Pre-Calculus, we expect you to be strong in Algebra 2, because the concepts learned in Algebra 2 are the foundation for all of Pre-Calculus. If you are struggling with factoring, solving a wide variety of equations, or understanding some basic function characteristics, it means you might not be totally ready to dive into Pre-Calculus quite yet, so it is important to review and practice those skills ahead of time.

Why can't I use a calculator?

While we will use calculators throughout the school year, sometimes they can become a crutch. It is important to exercise our brains and practice computations without a calculator to ensure our math mind stays strong!

If I do not *have* to do any of the practice problems – how will you check to see if I am prepared?

When we return to school in September, we will reserve some time during the first day of class to go over any questions you or your classmates may have about the material. This will give you some time to review and practice, but it will *not* be time for your teacher to re-teach every single topic to you. On our second day of class, you will have a **diagnostic test** – it won't count for a grade, but it's important for your teacher (and for you!) to see what you confidently know and what you still need to work on. When you get that test back, you'll have some time to review and correct it, and your teacher will be available to help outside of class. Then, towards the end of September you will have a second test on the material – the second test **will** count for a grade.

What if I have questions?

If you are having trouble with the material, or have a question that needs answering, there are plenty of ways to get help! Your math teachers will be checking their email regularly over the summer. You may not get an immediate response from them, but you can expect to hear from them within a few days. Old notes and textbooks or workbooks from Algebra 2 are also a wonderful place to look if you are struggling. Additionally, feel free to ask friends, classmates, family members, etc. for help as you are working! Collaborating and group study sessions can be a fantastic way to learn from others and reinforce the material.

Best of Luck! Don't hesitate to reach out with any questions!
-Ms. Jaffe (sjaffe@nya.org)

Essential Skills for Pre-Calculus

Algebraic Expressions

Factoring Polynomials	*Understand how to factor a wide variety of polynomials using strategies such as grouping and identifying the greatest common factor and recognizing common factoring patterns such as difference of squares and sum/difference of cubes.*
Operations with Exponents	*Perform operations with exponents, fully simplifying all answers.*
Operations with Radicals	*Perform operations with radicals, fully simplifying all answers.*
Operations with Complex Numbers	*Understand the pattern imaginary numbers follow when raised to a power. Perform operations with complex numbers, fully simplifying all answers.*
Operations with Rational Expressions	*Perform operations with rational expressions (including complex fractions), fully simplifying all answers.*

Solving Equations

Linear Equations and Inequalities	*Solve linear equations and inequalities, recognizing when they have either "no solution" or a solution of "all real numbers." For inequalities, know how to give answers in inequality notation, interval notation, and by graphing on a number line.*
Absolute Value Equations and Inequalities	*Use properties of equality to solve absolute value equations and inequalities, understanding how and when to separate into two cases, including checking for extraneous solutions.*
Quadratic Equations	*Understand how to solve quadratic equations using factoring, square roots, the quadratic formula, and completing the square, including being able to tell which method is most appropriate for a given equation.*
Radical Equations	*Solve radical equations regardless of the index, including checking for extraneous solutions.*
Rational Equations	*Solve rational equations, including checking for extraneous solutions.*

Characteristics of Functions

Domain and Range	*Given a set of ordered pairs or a graph, be able to identify the domain and range of the relation and determine if it represents a function. Determine the domain and range of functions using their equations and by visualizing or sketching the graph.*
Function Notation	*Evaluate functions given a problem in function notation. Determine whether you are being given the input (x) or the output (f(x)), and find the other value given an equation or a graph.*
Finding *x*- and *y*-intercepts	*Identify intercepts from a graph and be able to find the coordinates of x- and y-intercepts algebraically given an equation by substituting 0 in for either variable and solving.*

The skills listed here are from Algebra 2, but you are also expected to be fluent in topics learned in *previous* years of math, such as operations with fractions, basic exponent rules, divisibility, etc.

Essential Skills for Pre-Calculus

Factoring Polynomials

Always start by looking for the **Greatest Common Factor**.

For **binomials**: look for factoring patterns such as **difference of squares** or **sum/difference of cubes.**

- **Difference of Squares:** $a^2 - b^2 = (a+b)(a-b)$
- **Sum of Cubes:** $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- **Difference of Cubes:** $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

For **trinomials**: find a pair of numbers that multiplies to ac and adds to b. Use these numbers to break up the middle term, then factor by **grouping**. (If you have learned a different method that you prefer, that's fine too!)

For **4-term polynomials**: factor by grouping.

Practice: Fully factor each polynomial.

1. $16m^4 - 9n^2$
2. $1 - 216b^3$
3. $x^2 + 12x + 35$
4. $x^4 - 10x^2 + 24$
5. $10x^2 - 7x - 6$
6. $9k^2 + 18k + 8$
7. $4m^3 - 12m^2 + 3m - 9$
8. $24x^3 + 16x^2 - 60x - 40$
9. $-k^4 - 64k$
10. $7p^5 - 7p^4 - 42p^3$
11. $28n^3 - 12n^2 - 7n + 3$
12. $20x^2 - 115x - 30$

Operations with Exponents

- When two exponential expressions with the same base are **multiplied** together, their exponents get **added.**
- When two exponential expressions with the same base are **divided**, their exponents get **subtracted.**
- When one exponential expression is **raised to another exponent**, the **exponents get multiplied** together, with the exponent being distributed to each coefficient and variable in the base.
- An **exponent of negative one** results in the **reciprocal** of the base.
- Exponential expressions are considered fully simplified when each base only appears one time, all like terms are combined, and there are no negative exponents.

Practice: Fully simplify the given expressions.

1. $\left(-\frac{9}{8}p^{-3}q^7\right)(-32p^{-2}q^{-6})$
2. $\frac{(2ab^4c^{-3})^3}{-2c^{-2} \bullet -a^2b^3c^2}$
3. $\left(\frac{2a^2b^{-3} \bullet -b^2}{a^{-4}b^2}\right)^4$
4. $\frac{(-x^4y^{-3})^2}{-2x^{-4}y^0 \bullet 2x^3y^2}$
5. $(-9m^5n^4)^2 - 6m^8n^3(-7m^2n^5)$
6. $8x^{-10} + (-3x^{-4}y^{-2})\left(\frac{5}{x^6y^{-2}}\right)$

Essential Skills for Pre-Calculus

Operations with Radicals

Radicals are simplified by looking for **perfect squares** (or cubes, or fourths, etc.) that are **factors** of the expression under the radical sign (called the radicand). When combining like terms, look for radicals with the same **index** and the same **radicand** in order to combine. Radicals can be multiplied together when they have the same index – if they don't have the same index, you may want to write them in exponential form before multiplying! Much like complex numbers, you cannot leave a radical in the denominator of a fraction.

Practice: Fully simplify the given expressions.

1. $\sqrt{125a^6b^8c^3}$

2. $\sqrt[3]{-192p^6q^4}$

3. $-\sqrt[4]{24}+3\sqrt[4]{24}$

4. $-2x\sqrt[3]{32x^4}-3\sqrt[3]{4x^7}$

5. $\sqrt{15y^3}\bullet\sqrt{3y^2}$

6. $\left(-2-3\sqrt{2}\right)\left(\sqrt{2}-5\right)$

7. $\frac{4\sqrt{10}}{\sqrt{32}}$

8. $\frac{-5-2\sqrt{2}}{\sqrt{2}}$

9. $\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

10. $\frac{5+3\sqrt{2}}{-2-3\sqrt{3}}$

Practice: Fully simplify the given expressions and write your answer in both exponential *and* simplest radical form.

11. $k^{\frac{2}{3}}\bullet k^{\frac{5}{6}}$

12. $\frac{p^2}{p^{\frac{1}{4}}}$

13. $\left(m^{\frac{3}{4}}\right)^{\frac{20}{9}}$

14. $\left(\frac{x^{-\frac{1}{2}}}{x^{-5}}\right)^{\frac{7}{6}}$

Operations with Complex Numbers

The imaginary unit, i, is defined as $i=\sqrt{-1}$. When raising imaginary numbers to a power, it follows a specific pattern:

$i^1=i$ $\quad$ $i^2=\left(\sqrt{-1}\right)^2=-1$ $\quad$ $i^3=i\cdot i^2=-i$ $\quad$ $i^4=i^2\cdot i^2=1$

$i^5=i\cdot i^4=i$ $\quad$ $i^6=i^2\cdot i^4=-1$ $\quad$ $i^7=i^3\cdot i^4=-i$ $\quad$ $i^8=i^4\cdot i^4=1$

When simplifying expressions using complex numbers, terms that contain **imaginary numbers** do not combine (add or subtract) with terms that only contain **real numbers**. Much like radicals, you cannot leave imaginary numbers in the denominator of a fraction. If you have a binomial in the denominator, you will need to multiply by the **complex conjugate** in order to simplify.

Practice: Fully simplify the given expressions.

1. $\sqrt{-196}$

2. $\sqrt{-80}$

3. $\sqrt{-6}\bullet\sqrt{-10}$

4. i^{61}

5. $3i^{14}\bullet 5i^7$

6. $\left(i^5\sqrt{6}\right)^2\bullet(-4i)^3$

7. $(-7-2i)-(-6+4i)$

8. $(-15-3i)+(-4-8i)$

9. $(-1+4i)(11-6i)$

10. $(6-5i)^2$

11. $\frac{-7+2i}{6i}$

12. $\frac{4-10i}{3-i}$

Essential Skills for Pre-Calculus

Operations with Rational Expressions

Rational expressions are fractions that just happen to contain variables in the denominator. They follow all the same rules as fractions without variables:

- Multiply across the numerator and across the denominator
- Dividing a fraction is the same as multiplying by the reciprocal
- Adding and subtracting requires a common denominator
- Fully simplify by ensuring the numerator and denominator in your final answer do not have any common factors.

Practice: Fully simplify the given expressions.

1. $\frac{5m+9}{20m+36} \bullet \frac{2m^2+8m+8}{2m^2-4m-16}$

2. $\frac{6n+20}{7n^2+37n-30} \div \frac{6n+20}{35-49n}$

3. $\frac{x+2}{x-3} + \frac{5x}{5}$

4. $\frac{6}{k-6} + \frac{3k}{k+4}$

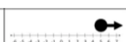
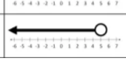
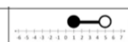
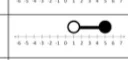
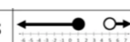
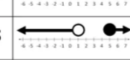
5. $2 - \frac{h+3}{10h+8}$

6. $\dfrac{\frac{16}{x+2} + \frac{4}{x+2}}{\frac{4}{3} - \frac{9}{x+2}}$

Linear Equations and Inequalities

In many cases, when solving an equation, we will find the *number* that the variable is equal to. However, there are some special cases of equations where we may have **no solution** or a solution of **all real numbers** instead. **Literal equations** are equations where instead of a number, your answer is an equation that has been rearranged to isolate a specific variable.

Solving inequalities is very similar to solving equations, but if you multiply or divide by a negative number, you need to reverse the direction of the inequality symbol. Solutions can be expressed in several different ways, such as **inequality notation**, **interval notation**, or by **graphing on a number line**.

$x \geq 5$		$[5, \infty)$
$x < 5$		$(-\infty, 5)$
$1 \leq x < 5$		$[1, 5)$
$1 < x \leq 5$		$(1, 5]$
$x \leq 1$ or $x > 5$		$(-\infty, 1] \cup (5, \infty)$
$x < 1$ or $x \geq 5$		$(-\infty, 1) \cup [5, \infty)$

Practice: Solve each equation.

1. $-5(m+1) - 7 = -2m + 9$

2. $\frac{8}{3}\left(3x - \frac{19}{4}\right) = -42$

3. $-3(4-8p) + 6 = 4(6p-1) - 2$

4. $-\frac{4}{3}(12v - 18) + 32 = -8(7 + 2v)$

5. $l = \frac{x \bullet 2\pi r}{360}, solve\ for\ x$

6. $c = \frac{1}{3}(a^2 + b), solve\ for\ a$

Practice: Solve each inequality. You should be comfortable giving your answer in inequality notation, interval notation, and by graphing.

7. $-124 > -4(1-6x)$

8. $-3 + 4(4v - 6) \geq -107$

Essential Skills for Pre-Calculus

Absolute Value Equations and Inequalities

Absolute value represents the distance from zero, without considering the direction. This causes absolute value equations and inequalities to typically split into **two cases** in order to properly represent that there are always two numbers (a positive and a negative number) that have the same *distance from zero*. When solving absolute value inequalities, don't forget to reverse the direction of the inequality sign when needed!

Absolute value (distance) cannot be negative – if you isolate an absolute value expression and it equals a negative number, the equation has no solution.

Absolute value equations can have **extraneous solutions** – always be sure to check your answers to see if any are extraneous.

Practice: Solve each equation.

1. $6|p-2|-4=8$

2. $-3|8x+1|-3=-24$

3. $4r-3=|2r+9|$

4. $\frac{1}{2}|9c+3|-5=3c-11$

Practice: Solve each inequality. You should be comfortable giving your answer in inequality notation, interval notation, and by graphing.

5. $\frac{1}{10}|7x+7|>4$

6. $-5|1-6k|-10\geq -95$

Quadratic Equations

There are five methods that can be used to solve quadratic equations: graphing, factoring, square roots, completing the square, and the quadratic formula. You should be familiar and comfortable with all five methods. Completing the square and the quadratic formula will work for all cases, whereas graphing, factoring, and square roots only work sometimes. Quadratics can have real (rational or irrational) or complex solutions, and solutions should always be given in simplified form.

Practice: Solve each equation by factoring.

1. $5n^2-n-6=0$

2. $15y^2+7=26y$

Practice: Solve each equation using square roots.

3. $-\frac{1}{4}k^2+17=7$

4. $3w^2+9=-11$

Practice: Solve each equation by completing the square.

5. $v^2+16v+48=-5$

6. $7w^2-14w-85=-8$

Practice: Solve each equation using the quadratic formula.

7. $4n^2+6n+6=0$

8. $r^2-10r-2=0$

Essential Skills for Pre-Calculus

Radical Equations

To solve a radical equation, isolate the radical expression and then use the inverse operation (square both sides if you have square roots, cube both sides if you have cubed roots, etc.). When you are working with radicals with an even index (square roots, fourth roots, etc.), it is essential to check for extraneous solutions, as taking an even root of a negative number results in a complex number.

Practice: Solve each equation.

1. $\sqrt{4x+1} = \sqrt{5x}$
2. $\sqrt[4]{\frac{a}{6}} = \sqrt[4]{7-a}$
3. $\sqrt[3]{-1-13m} + 8 = 12$
4. $4\sqrt{2x-10} + 2 = 10$
5. $2 = \sqrt{x+14} - x$
6. $\sqrt{-1-2p} = p$

Rational Equations

Rational equations consist of fractions, so solving them requires a strong understanding of simplifying fractions – adding and subtracting fractions requires a common denominator, and fractions are simplified by cancelling common factors. Rational equations can have extraneous solutions, so it is important to ensure your solution doesn't require you to divide by zero when checking your work!

Practice: Solve each equation.

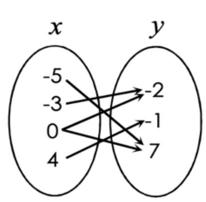
1. $\frac{2}{2w-3} = \frac{w-2}{3w-7}$
2. $\frac{3}{2n} - 3 = \frac{1}{2n}$
3. $\frac{x-2}{2x} + \frac{x^2+7x+6}{2x^2} = \frac{3}{2}$
4. $\frac{4}{k^2+2k} + \frac{6}{k} = \frac{k+3}{k^2+2k}$
5. $\frac{1}{m^2-25} + \frac{1}{m-5} = \frac{m+6}{m+5}$
6. $\frac{a+4}{3} - \frac{4a^2-28a+24}{3a-9} = \frac{a^2+3a-10}{a-3}$

Domain and Range

Relations can be expressed several ways: mapping diagrams, graphs, sets of ordered pairs, equations, and tables. They can be **continuous** or **discrete**. The **domain** of a relation is the set of all x-values. The **range** is a set of all y-values. The domain and range for discrete relations are represented using a set of numbers. For continuous relations, use either set or interval notation. A **function** is a relation that contains no repeating x-values.

Practice: Find the domain and range of each relation and determine if the relation is a function. For continuous relations, give your domain and range in both interval and set notation.

1.



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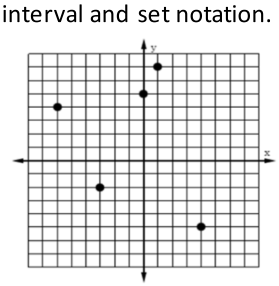

3.

$\{(-3,-9), (-2,-1), (-2,-6), (-1,-9)\}$

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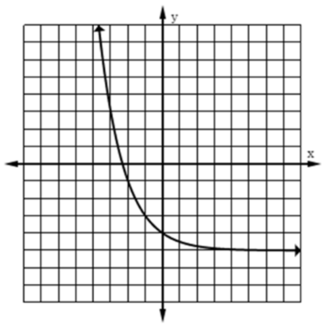

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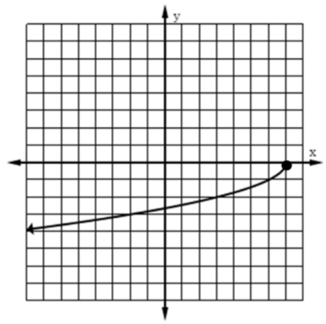

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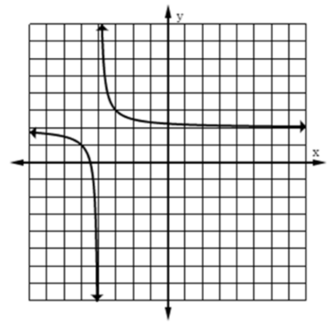

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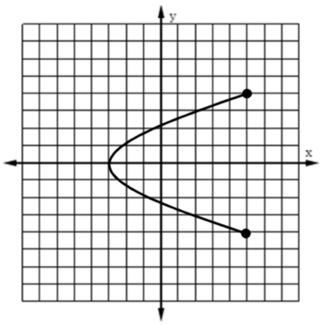

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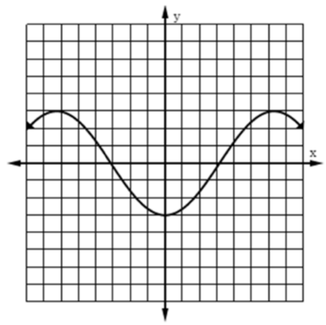

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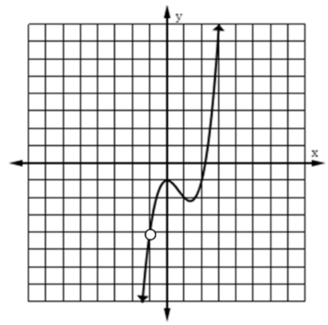

Practice: Use what you learned about different functions in Algebra 2 to determine the domain and range of each function. It might help you to try to visualize what the graphs look like! Give your answers in both interval and set notations.

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10. $f(x) = x$
11. $f(x) = 3x + 7$
12. $f(x) = |x|$
13. $f(x) = \frac{1}{2}|x| - 6$
14. $f(x) = x^2$
15. $f(x) = -x^2 + 2$
16. $f(x) = \sqrt{x}$
17. $f(x) = \sqrt{x+2} + 9$
18. $f(x) = \sqrt[3]{x}$
19. $f(x) = -\sqrt[3]{x+3}$
20. $f(x) = \frac{1}{x}$
21. $f(x) = \frac{3}{x-1} - 5$

Function Notation

Function notation is used to make it clear which function is being referred to, which can be especially helpful when looking at multiple functions together. It also allows for shorthand. Rather than saying "plug 3 in for x and evaluate", we can simply say "f(3)", which means the same thing.

Practice: Evaluate each function for the given value.

1. $f(x) = -x^3 + 2x^2 + 10$; $f(3)$
2. $g(x) = |10 - x^2|$; $g(5)$
3. $p(x) = \frac{x^2+4}{2x}$; $p(-8)$
4. $h(x) = 4 \bullet 4^x$; $h(-3)$
5. $j(x) = -x^2 - 3x + 10$; $j(3x + 2)$
6. $k(x) = \frac{2x+15}{x+9}$; $k(-3w)$

Finding *x*- and *y*-Intercepts

When looking at a graph, we can identify intercepts by looking for the place where the graph crosses the *x*- or *y*-axis. Algebraically, we can find intercepts (without a graph) by substituting 0 for one variable and solving for the other. To find an *x*-intercept, substitute 0 for *y* and solve for *x*. To find the *y*-intercept, substitute 0 for *x* and solve for *y*. Functions can have *infinite* x-intercepts, but they can only have at most one y-intercept. They also may not have any intercepts at all! X-intercepts are also called zeros, solutions, and roots.

We can write intercepts either by saying what the variable equals, or by using an ordered pair. For example, an *x*-intercept of 4 could be written as x=4, or as the ordered pair (4, 0).

Practice: Find the x- and y-intercepts of each function **algebraically**. Express any irrational answers in simplest radical form.

1. $f(x) = -|x-3| + 5$
2. $f(x) = |3x-5| - 2$
3. $f(x) = 2x^2 - 7x - 4$
4. $f(x) = x^3 - 10x^2 + 24x$
5. $f(x) = \sqrt{x+6} - 3$
6. $f(x) = \frac{1}{2}\sqrt{x-4}$
7. $f(x) = \frac{1}{x+1} - 3$
8. $f(x) = \frac{-2}{x+4} + 5$