Algebra 2 Summer Homework

Welcome to Algebra 2!

While we certainly hope you have a good summer filled with fun and enriching activities, as well as plenty of time relaxing with family and friends, we also want to ensure everyone is entering Algebra 2 on equal footing, and with a solid foundation that will help them start the school year on a positive note. For that reason, we have compiled a list of topics we have identified as the *most important* content from prior years. **Over the summer, your job is to make sure you are comfortable with these skills and topics without the use of a calculator.**

What is my assignment?

The work you do over the summer will be different for everyone. As you look at the topics listed, if you feel comfortable and confident with all the material, you may just choose to do a few practice problems from each topic to brush up at the end of August. If the topics listed seem unfamiliar, or you cannot even remember learning them in Algebra 1, then you will have more practice to do over the summer to ensure you are ready for Algebra 2. In short – do as much or as little practice as needed (**and if you need or want even more than what is included here, feel free to reach out to your teacher**!), but please make sure you are comfortable with the material by the time school starts in September.

Why do I have to do this?

In math, just like in foreign languages, it is essential to review past material and keep it fresh in your brain, otherwise, it gets forgotten. Past material is also the building blocks of the new material we'll be learning! This means that by the time you get to Algebra 2, we expect you to be strong in Algebra 1, because the concepts learned in Algebra 1 are the foundation for all of Algebra 2. If you are struggling with basic operations, solving equations, and working with linear functions, it means you might not be totally ready to dive into Algebra 2 quite yet, so it is important to review and practice those skills ahead of time.

Why can't I use a calculator?

While we will use calculators throughout the school year, sometimes they can become a crutch. It is important to exercise our brains and practice computations without a calculator to ensure our math mind stays strong!

If I do not *have* to do any of the practice problems – how will you check to see if I am prepared?

When we return to school in September, we will reserve some time during the first day of class to go over any questions you or your classmates may have about the material. This will give you some time to review and practice, but it will *not* be time for your teacher to re-teach every single topic to you. On our second day of class, you will have a **diagnostic test** – it won't count for a grade, but it's important for your teacher (and for you!) to see what you confidently know and what you still need to work on. When you get that test back, you'll have some time to review and correct it, and your teacher will be available to help outside of class. Then, towards the end of September you will have a second test on the material – the second test **will** count for a grade.

What if I have questions?

If you are having trouble with the material, or have a question that needs answering, there are plenty of ways to get help! Your math teachers will be checking their email regularly over the summer. You may not get an immediate response from them, but you can expect to hear from them within a few days. Old notes and textbooks or workbooks from Algebra 1 are also a wonderful place to look if you are struggling. Additionally, feel free to ask friends, classmates, family members, etc. for help as you are working! Collaborating and group study sessions can be a fantastic way to learn from others and reinforce the material.

Best of Luck! Don't hesitate to reach out with any questions!

-Ms. Jaffe (sjaffe@nya.org) and Ms. Zanetell (jzanetell@nya.org)

Essential Skills for Algebra 2

Computation

The Real Number System	*Understand the difference between each classification of number.*
Numeric Properties	*Understand numerical concepts and how they can apply to different math problems.*
Order of Operations	*Be comfortable fully simplifying multi-step expressions, including expressions including any of the components listed below in this table.*
Basic Exponents and Exponent Rules	*Understand what happens when raising negative numbers to even or odd powers. Simplify exponential expressions using the laws of exponents.*
Simplifying Square Roots	*Know the first 15 perfect squares and understand how to write square roots in simplest radical form.*
Fractions	*Be comfortable and fluent with all operations relating to fractions. Give all answers as fully simplified improper fractions unless told otherwise.*

Algebra Basics

Algebraic Properties	*Understand and be able to apply properties of equality to expressions and equations with variables.*
Combining Like Terms/Simplifying Expressions	*Differentiate between like and unlike terms when working with algebraic expressions and accurately combine them to simplify.*
Evaluating Expressions	*Given an algebraic expression, substitute given values and simplify.*
Solving Multi-Step Equations	*Use the properties of equality to isolate a variable and solve an equation. Confidently recognize when equations have either "no solution" or a solution of "all real numbers."*
Solving Multi-Step Inequalities	*In addition to solving, know how to give answers in inequality notation, interval notation, and by graphing on a number line.*
Solving Literal Equations	*Given equations with multiple variables, solve for any indicated variable.*
Solving Absolute Value Equations and Inequalities	*Use properties of equality to solve absolute value equations and inequalities, understanding how and when to separate into two cases.*
Translating Word Problems to Algebra	*Understand key words that indicate each mathematical operation, represent unknowns with variables, and be able to evaluate if an answer makes sense given the context of the problem.*

Linear Equations

Slope-Intercept Form	*Understand the different components of the form. Comfortably manipulate any linear equation to put it into slope-intercept form.*
Standard Form	*Understand the different components of the form. Comfortably manipulate any linear equation to put it into slope-intercept form.*
Point-Slope Form	*Given either the slope and a point, or two points, write an equation in point-slope form and simplify it to slope-intercept form when asked.*
Graphing Linear Equations	*Comfortably graph all linear equations, regardless of which equation form you are given to start with.*
Finding *x*- and *y*-intercepts	*Identify intercepts from a graph and be able to find the coordinates of x- and y-intercepts algebraically given a linear equation.*
Calculating Slope	*Calculate the slope of a line when given two points or a graph.*
Parallel and Perpendicular Lines	*Given the equation of a line, write a second equation that is parallel or perpendicular at a certain point to the first.*

Essential Skills for Algebra 2

The Real Number System and Numeric Properties

- Understand the difference between Rational and Irrational Numbers
- Understand the sub-classifications of rational numbers: Integers, Whole Numbers, Natural Numbers
- Understand and be able to use the theories behind the following properties of addition and multiplication: commutative, associative, identity, inverse
- Understand and be able to use the distributive property

Order of Operations

1. Grouping Symbols – parentheses, brackets, braces, absolute value, and even fraction bars are all some examples of grouping symbols
2. Exponents – this includes any radicals as well
3. Multiplication and Division – start at the left and work your way right, just like reading
4. Addition and Subtraction – start at the left and work your way right, just like reading

Practice: Fully simplify each expression.

1. $\frac{7-|10-12|+5^2}{9-2^2}$
2. $\frac{\left(2-3^2\right)^2+\sqrt{81}}{10-6\div 3}$
3. $\frac{\left[15-\left(|-6+2|+\sqrt{9}\right)\right]^2}{3-3^3}$
4. $\frac{(9-15)^2\cdot(7-2^2)}{\sqrt{49}+5}-|-5|$

Additional Practice: Fully simplify the exponential expression

5. -4^2
6. $(-4)^2$
7. -4^3
8. $(-4)^3$
9. $2m^3 \bullet 2m^2$
10. $3k^2 \bullet k$
11. $(2x^3)^4$
12. $\frac{b}{2b^2}$

Additional Practice: Fully simplify the radical expression

13. $\sqrt{144}$
14. $\sqrt[3]{-27}$
15. $\sqrt{x^6}$
16. $\sqrt{96}$

Additional Practice: Perform the indicated operations and fully simplify the resulting fractions. Leave your answers as improper fractions, not mixed numbers or decimals!

17. $\frac{3}{4}+\frac{2}{5}$
18. $7-\frac{1}{4}$
19. $\frac{7}{4}+\frac{5}{8}$
20. $\frac{3}{7}-3$
21. $1+\frac{4}{3}$
22. $\frac{3}{2}+\frac{11}{6}-\frac{5}{4}$
23. $\frac{1}{4}\cdot\frac{5}{3}$
24. $\frac{13}{4}\div\frac{1}{2}$
25. $\frac{15}{8}\cdot\frac{7}{6}$
26. $\frac{13}{7}\cdot\frac{14}{11}$
27. $\frac{9}{4}\div 5$
28. $\frac{1}{2}\div\frac{3}{4}\cdot\frac{2}{7}$

Essential Skills for Algebra 2

Combining Like Terms/Simplifying Expressions

A term is either a single number/variable, or numbers and variables multiplied together. Terms are separated by addition or subtraction signs. We combine like terms by adding or subtracting as indicated. "Like Terms" are terms that have the *exact same variables* and the *exact same exponents*.

Practice: Fully simplify each expression by distributing and/or combining like terms.

1. $16x - 11 - 7x + 10$
2. $16(c - 2) - (8c + 1)$
3. $3(3a - 5b + 1) - (a - 7b) + 4$
4. $-2(x^2 + y) + 3(x + y) - x^2$

Evaluating Expressions

When we **evaluate**, we are being asked to find a numerical answer. This typically involves substituting numbers into an algebraic expression and then simplifying to get a final numerical answer. Sometimes, you may be asked to substitute in a different *variable,* rather than a number. In these cases, your final answer will likely still include variables in it.

Practice: Evaluate each algebraic expression using the given numbers.

1. $\frac{a \cdot b^2}{c+12}$ $(if\ a = 3,\ b = -4,\ and\ c = 8)$
2. $-x^2 + 7(y + 2) - 1$ $(if\ x = 5\ and\ y = -9)$
3. $\frac{1}{5}\left(3r - 4s + \frac{3}{2}\right)$ $\left(if\ r = 5\ and\ s = \frac{2}{3}\right)$
4. $\frac{2}{5}(a + 2b)^2$ $(if\ a = 3\ and\ b = -4)$

Solving Multi-Step Equations

1. Distribute
2. Fully simplify each side by combining like terms
3. Move all the variables to the same side of the equation
4. Solve by isolating the variable using inverse operations

In many cases, when solving an equation, we will find the *number* that the variable is equal to. However, there are some special cases of equations where we may have **no real solution** or a solution of **all real numbers** instead.

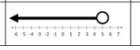
Practice: Solve each equation.

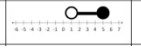
1. $6(b - 8) = -84$
2. $-56 = -8(m - 5) - 6(5m - 3)$
3. $5(k + 3) + 2(6k + 6) = -41$
4. $-2v + 17 = 3(-1 + v) - 5$
5. $7(1 - 2v) = -(1 + 6v)$
6. $-4\left(\frac{10}{3}m - 2\right) = -42$
7. $-42 = -\frac{7}{2}\left(\frac{5}{2}x + 2\right)$
8. $\frac{x+14}{7} = \frac{2x-5}{3}$
9. $-2(v - 2) = -3 - 2v$
10. $4(k - 8) = -32 + 4k$

Essential Skills for Algebra 2

Solving Multi-Step Inequalities

Solving inequalities is very similar to solving equations, but if you multiply or divide by a negative number, you need to reverse the direction of the inequality symbol. Solutions can be expressed in several different ways, such as **inequality notation**, **interval notation**, or by **graphing on a number line**.

$x \geq 5$		$[5, \infty)$
$x < 5$		$(-\infty, 5)$

$1 \leq x < 5$		$[1, 5)$
$1 < x \leq 5$		$(1, 5]$

$x \leq 1$ or $x > 5$		$(-\infty, 1] \cup (5, \infty)$
$x < 1$ or $x \geq 5$		$(-\infty, 1) \cup [5, \infty)$

Practice: Solve each inequality. Express your answer in inequality notation, interval notation, *and* as a graph on a number line.

1. $114 \leq -3(6p - 1) + 3$
2. $116 \leq 4(1 + 4x)$
3. $7(2 - 4x) + 3 > 31 - 4x$
4. $-6n + 5 > 2n - 7n + 14$

Solving Literal Equations

A literal equation is an equation that has two or more variables. Because there are multiple variables, we can express these types of equations in different ways by solving for different variables. **Formulas** are some of the most common examples of literal equations.

Practice: Solve each literal equation by *isolating* the indicated variable.

1. $mx = p - n; \quad solve\ for\ x$
2. $g = -b + \frac{c}{a}; \quad solve\ for\ a$
3. $-\frac{4a}{3} = -v + 3w; \quad solve\ for\ a$
4. $4t - 1 = 9sr; \quad solve\ for\ t$

Solving Absolute Value Equations and Inequalities

Absolute value represents the distance from zero, without considering the direction. This causes absolute value equations and inequalities to typically split into **two cases** in order to properly represent that there are always two numbers (a positive and a negative number) to have the same *distance from zero*. When solving absolute value inequalities, don't forget to reverse the direction of the negative sign when needed!

Practice: Solve each absolute value equation or inequality. For inequalities, express your answer in inequality notation, interval notation, *and* as a graph on a number line.

1. $|v - 5| = 8$
2. $-6\left|\frac{n}{4}\right| = -15$
3. $\frac{|9-2n|}{6} = 1$
4. $3|6x + 5| + 10 = 103$
5. $|2x - 3| = x$
6. $|x - 2| = 3x + 1$
7. $|n - 10| + 3 \leq 19$
8. $|v + 3| + 2 > 14$

Essential Skills for Algebra 2

Translating Word Problems to Algebra

When reading through word problems, it is helpful to first **define your variable(s)** by identifying what it is you are trying to solve or find. Then, be on the lookout for key words that can help you identify what operations to use in your equation.

- **Addition**: sum, total, more than, added, increased, plus
- **Subtraction**: less, minus, decreased by, difference, less than
- **Multiplication**: product, times, of, per, multiplied
- **Division**: quotient, divided by, divided into

Practice: For each scenario, write and solve an algebraic expression or equation that models the information.

1. If 3 times a number is increased by 22, the result is 14 less than 7 times the number. What is the number?
2. One complementary angles measures 18 less than five times the other angle. What is the measure of each?
3. In 8 years Jamie will be three times as old as she is now. How old is she now?

Slope-Intercept Form

Commonly referred to with the equation $y = mx + b$, slope-intercept form is one of the three forms of equations. An equation is in slope-intercept form if **y is isolated,** and all parts of the equation have been fully simplified. Slope-intercept form is helpful because it tells us the **slope** (m) of the line, as well as the **y-intercept** (b).

Practice: Write the equation of each line in slope-intercept form.

1. $slope = -\frac{2}{3};\ y - int = 3$
2. $slope = \frac{1}{2};\ y - int = -5$
3. $11x + 8y = -32$
4. $3x - 2y = -12$
5. $x + 3y = 24$
6. $2x + y = 2$

Standard Form

The standard form of a linear equation is $Ax + By = C$. Standard form follows several rules:

- The x and y variable are on the same side of the equation, with the constant on the opposite side
- There are no fractions or decimals at all in the equation – every number is an integer
- "A" needs to be a positive number
- "A" "B" and "C" need to be *relatively prime* – this means there are no common factors between the three

Practice: Put each equation into standard form.

1. $y = 3x - 2$
2. $y = -\frac{7}{4}x + 2$
3. $y = \frac{3}{4}x - 1$
4. $y = -\frac{1}{2}x - 6$

Essential Skills for Algebra 2

Point-Slope Form

Point-slope form is used when we know the slope and a coordinate on the line that isn't the y-intercept, or when we are just given two points, and can then calculate the slope using the slope formula.

Point-Slope Form is written as $y - y_1 = m(x - x_1)$

- m is the slope of the line
- (x_1, y_1) is the coordinate you are given

Practice: First, write an equation in point-slope form using the given information. (If you are only given two points, you will need to first find the slope using the slope formula!) Then, distribute and isolate y to put the equation into slope-intercept form.

1. $through: (3, -5), slope = 1$
2. $through: (-2, -2), slope = \frac{3}{2}$
3. $through: (4, 3), slope = \frac{4}{9}$
4. $through: (5, -4)\ and\ (1, -1)$

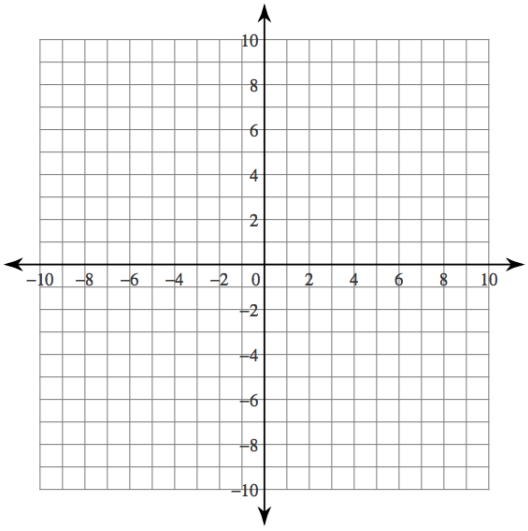
Graphing Linear Equations

Linear equations are graphed on the coordinate plane. We can graph any linear equation no matter what form it is in, but it may feel most comfortable to first put all equations into slope-intercept form so you can easily see the y-intercept and slope.

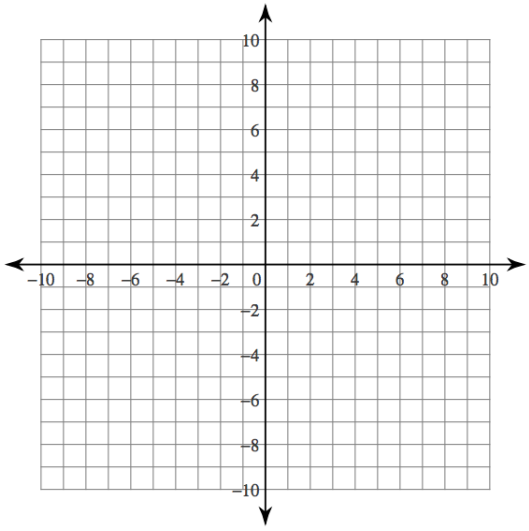
Most lines that we graph are diagonal, with either a positive or negative slope. Some lines, however, will be either horizontal (slope of 0) or vertical (undefined slope). This happens when we have an equation with only one variable. Equations with only an x-variable result in a vertical line. Equations with only a y-variable result in a horizontal line.

Practice: Graph each linear equation.

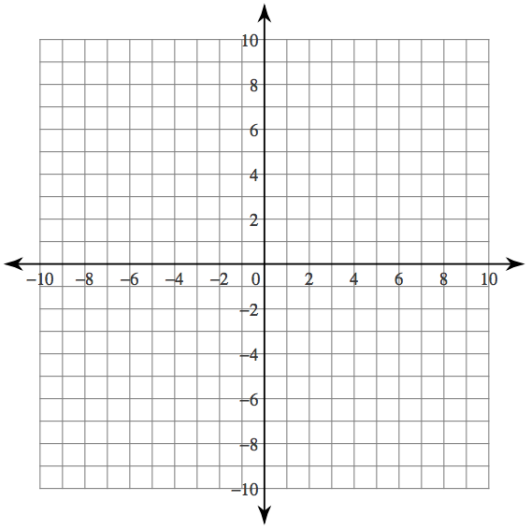
1. $y = -x - 3$



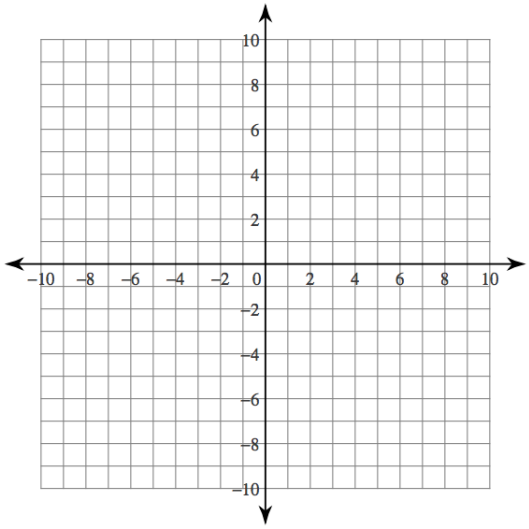
2. $y = 2$



3. $4x - 3y = -6$



4. $x + 5y = -20$



Finding *x*- and *y*-intercepts

When looking at a graph, we can identify intercepts by looking for the place where the graph crosses the *x*- or *y*-axis. Algebraically, we can find intercepts (without a graph) by substituting 0 for one variable and solving for the other. To find an *x*-intercept, substitute 0 for *y* and solve for *x*. To find the *y*-intercept, substitute 0 for *x* and solve for *y*.

We can write intercepts either by saying what the variable equals, or by using an ordered pair. For example, an *x*-intercept of 4 could be written as x=4, or as the ordered pair (4, 0).

Practice: Given each linear equation, find the *x*- and *y*-intercepts algebraically.

1. $9x + 4y = 16$
2. $7x - 5y = -30$

Calculating Slope

Slope is a **constant rate of change.** We can find the slope of a line by calculating the rate of change between any two points. On a graph, we can look for how much the graph is rising (or falling) between two points and divide that by how far over we go over between those same two points.

To find slopes algebraically, we use the **slope formula**, which uses 2 coordinates - (x_1, y_1), (x_2, y_2)

$$slope\ (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

Practice: Find the slope of the line connecting the two given points.

1. $(2,\ 1)\ and\ (0,\ 2)$
2. $(0,\ -2)\ and\ (-5,\ -1)$
3. $(-4,\ 5)\ and\ (1,\ -2)$
4. $(3,\ 7)\ and\ (8,\ 1)$

Essential Skills for Algebra 2

Parallel and Perpendicular Lines

Parallel lines (∥) are coplanar lines that never touch, which can only happen if they have the **same slope** but different **y-intercepts. Perpendicular lines (⊥)** are lines that intersect each other to form a $90°$ angle, which happens when two lines have slopes that are **opposite reciprocals** of each other.

Examples of **Opposite Reciprocals**:

$$-\frac{2}{3} \text{ and } \frac{3}{2} \qquad 5 \text{ and } -\frac{1}{5} \qquad \frac{1}{6} \text{ and } -6$$

If you are asked to determine whether lines are parallel or perpendicular (or neither), you can do so by comparing the slopes of the lines and making sure that lines that have the same slope don't *also* share the same y-intercept.

If you are given one equation and asked to write the equation of a parallel or perpendicular line that passes through a specific point, once you identify the correct slope, you can use **point-slope form** to write your new equation, and then distribute and simplify to get an equation in **slope-intercept form** if necessary.

Practice: Determine whether the lines are parallel, perpendicular, or neither.

1. $9x - 4y = -12 \text{ and } y = -\frac{9}{4}x - 3$
2. $5x - 2y = 10 \text{ and } y = -\frac{2}{5}x + 3$

Practice: Given the equation of one line, write an equation in slope-intercept form for a second line that satisfies the given criteria.

3. $through\text{: } (-1,\ 2),\ \parallel\ to\ y = -\frac{7}{4}x - 1$
4. $through\text{: } (-2,\ 0),\ \parallel\ to\ y = \frac{5}{2}x - 2$
5. $through\text{: } (4,\ 3),\ \perp\ to\ y = -\frac{2}{3}x + 4$
6. $through\text{: } (-2,\ 1),\ \perp\ to\ y = \frac{1}{3}x + 1$