Welcome to Algebra 1

I hope you have a great summer filled with fun and enriching activities and plenty of time relaxing with family and friends! To ensure everyone is entering Algebra 1 next fall with a solid Pre-Algebra foundation, I have compiled a list of the essential pre-Algebra concepts required to find success in Algebra I. Over the summer, your job is to make sure you are comfortable with these skills without the use of a calculator.

What is my assignment?

The amount of work required over the summer will be different for everyone. As you look at the topics listed in this document, you may feel comfortable and confident with certain skills and therefore only need to brush up on topics by completing a few practice problems. Perhaps some of the topics listed seem less familiar, or you may not remember having learned them in Pre-Algebra; if this is the case, you will need to spend more time over the summer preparing for Algebra 1. In short – do as much or as little practice as needed to ensure you are comfortable with the material by the time school starts in September. If you need and/or want additional resources beyond those included here, feel free to reach out with your request(s)!

Why do I have to do this?

Math, like foreign languages, is extremely cumulative in nature. Therefore, is essential to review past material to keep it fresh in your brain, so that it is not easily forgotten. This prior knowledge is essential to mastering the content we'll be learning next year! This means that we expect you to be proficient with the Pre-Algebra concepts, as they are foundational for all of Algebra 1. If you are struggling with the topics outlined below, you might not be fully ready to dive into Algebra 1, so reviewing and practicing ahead of the school year is important.

Why can't I use a calculator?

While we use calculators throughout the school year, they can sometimes become a crutch. It is important to exercise our brains and practice computations without a calculator to ensure our math mind stays strong!

If I do not have to do the practice problems – how will you know if I am prepared?

When we return to school in September, the first day of class is reserved for introducing ourselves and answering any questions you or your classmates may have. On the second day of class, you will take a diagnostic test – while it will not count as a graded assessment, it will inform us about the skills you have mastered and those that you may still need to work on. Once the test is corrected and returned, you will be able to review any mistakes and identify any gaps that require further attention before taking a second test on the material at the end of September (that will be graded).

What if I have questions?

If you are experiencing any trouble with the material or have a question that you need to be answered, there are plenty of resources at your disposal! Past notes, textbooks, and workbooks from Pre-Algebra are a great place to start if you are struggling. Additionally, feel free to ask friends, classmates, family members, etc. for help as you are working! Collaborating and group study sessions can be a fantastic way to learn from others and help reinforce the material. I will also be checking my email periodically over the summer. Although you may not receive an immediate response, you can expect a reply within a couple of weeks.

Best of Luck! Don't hesitate to reach out with any questions!

~Ms. Violette (jviolette@nya.org)

Essential Skills from Pre-Algebra Required for Algebra I

Fact Fluency	*Be proficient at recalling basic arithmetic facts ($+,\ -,\ \times, and\ \div$) with accuracy and from memory (without the need for counting or using a calculator).*
Operations with Integers	*Be able to perform mathematical operations ($+,\ -,\ \times, and\ \div$) with $\pm$ integers.*
Absolute Value	*Understand that absolute value is a mathematical concept that represents the distance between a number and zero on the $number\ line$, and is always a positive value. Be able to evaluate expressions and equations that contain absolute values.*
Order of Operations	*Know the correct order in which to perform arithmetic operations to simplify multi-step expressions.*
Evaluating Expressions with Variables	*Be comfortable replacing variables in an expression with their corresponding values and simplifying to obtain a numerical result.*
Combining Like Terms	*Be aware that combining like terms involves adding or subtracting terms that have the same variable(s) and exponent(s), to simplify expressions making them easier to work with.*
Distributive Property	*Simplifying multiplication problems by distributing a number or term to each term inside parentheses.*
Translate Word Sentences into Expressions / Equations / Inequalities	*Use your knowledge of mathematical vocabulary, operations, and symbols to accurately represent given information as a mathematical expression, equation, or inequality.*
Solving Linear Equations	*Solving linear equations means finding the value of an unknown variable that makes the equation true. **One-step equations** require only one math operation to solve, while **two-step equations** require two operations.*
Inequalities	*Be able to solve and graph inequalities (finding the set of values that make an inequality true and representing the solution on a number line).*

I. Fact Fluency

Fact fluency is the ability to recall basic arithmetic facts (addition, subtraction, multiplication, and division) quickly and accurately.

It involves the development of automaticity, which means that the recall of math facts becomes almost automatic and does not require conscious effort or attention.

Fact fluency is an important foundation for more complex mathematical operations and can improve problem-solving abilities and overall mathematical performance.

Suggestions:

1. Practice regularly: Consistent practice is key to improving fact fluency. Set aside a few minutes each day to practice math facts using flashcards, games, or online resources.
2. Focus on weaker areas: Identify the areas where you struggle the most and focus on improving them. Spend extra time practicing these areas until you feel more confident.
3. Use a variety of strategies: Use a variety of strategies to practice math facts, such as using visual aids, chunking, and breaking down larger problems into smaller ones. This will help to reinforce your understanding of the underlying concepts and make it easier to recall the facts quickly and accurately.

Practice:

1. $9 + 12$
2. 3×9
3. $45 \div 5$
4. $8 * 7$
5. $20 - 8$
6. $6 + 17$
7. $6 \cdot 8$
8. $88 \div 11$
9. $12 * 8$
10. 12×0
11. $49 \div 7$
12. $15 - 8$

II. Operations with Integers

While operations with integers and fact fluency are related in that they both involve mathematical calculations, they are different in terms of the skills and knowledge required. Operations with integers require a deep understanding of mathematical concepts and the ability to apply them accurately and efficiently, while fact fluency requires memorization and recall of basic mathematical facts.

Practice:

1. Start with simple problems involving addition, subtraction, multiplication, and division of small integers, such as 2 + 3, 7 - 5, 4 x 2, and 10 ÷ 5.
2. Gradually increase the difficulty of the problems by working with larger integers and negative values.
3. Use online resources, such as math games and quizzes, to practice your skills and identify areas where you may need additional practice.
4. Finally, check your answers using a calculator or by working backward from the answer ensuring that you correctly solved the problem.

Practice:

1. $82 + (-68)$
2. $-17 + 35$
3. $96 \div (-8)$
4. -12×7
5. $3(-82)$
6. $-306 \div (-34)$
7. $-435 \div 5$
8. $-98 - (-21)$
9. $53 - -34$
10. $-49 \div -7$

III. Absolute Value

Absolute value is a mathematical concept that denotes the distance of a number from zero, regardless of whether the number is positive or negative.

It is represented by enclosing the number in vertical bars or using the mathematical symbol "$|x|$", where "x" is the number whose absolute value is being determined.

The result of the absolute value of a number is always a non-negative value, as it only represents the distance from zero without considering direction.

Examples: $|-10| = 10$ $|7 - 15| = 8$ $|-2 + 8| = 6$

Practice: Simplify each absolute value expression below.

1. $|21|$
2. $|-86.5|$
3. $|-1| + |1|$
4. $|13 - 18|$
5. $|28| - |-25|$
6. $|-5 * 9|$
7. $|23 - 42|$
8. $|-18| - |-5|$

IV. Order of Operations

Order of operations is a set of rules that dictate the order in which mathematical operations should be performed. The concept is important because it ensures that mathematical expressions are evaluated consistently and correctly.

The order of operations is usually remembered using the acronym PEMDAS (*or GEMDAS):

Parentheses*:** Do operations inside parentheses first. *This first step includes all ***Grouping symbols, therefore you need to remember to evaluate any expressions inside grouping symbols (parentheses, brackets, and braces) from the innermost set to the outermost set.

***Exponents*:** Do any exponents or radicals next.

***Multiplication and Division*:** Evaluate multiplication and division in order from left to right.

***Addition and Subtraction*:** Evaluate addition and subtraction in order from left to right.

Example A:

$[3 + (5 - 2)^2] \div 4 + 5$	First, evaluate inside the brackets starting with inside the parentheses
$[3 + 3^2 \ ] \div 4 + 5$	Continue evaluating inside the brackets by applying the exponent
$[3 + 9 \ ] \div 4 + 5$	Finish evaluating inside the brackets by adding
$12 \div 4 + 5$	Next, divide
$3 + 5$	Finally, add
8	

Practice: Fully simplify each expression.

1. $17 - 2^5 \div 4 + 6$
2. $\frac{(7-2^2)+17}{-14+2\cdot 5}$
3. $-|-9 - 10| + 12^2 \div 6 \cdot 3$
4. $|-7^2 + 3^2 \cdot 4| + 18 \div 2 \cdot 5$
5. $\frac{5^2-3^4}{(2^3-5)(6^2\div 9)}$
6. $20^2 \div \{[3(5-9)^2] + 2\}$
7. $\frac{12[30-(9+4^2)]}{|-10|-|-6|}$
8. $\frac{84}{12} + (5 - 7^2) - 72 \div 6 \cdot 2$

V. Evaluating Expressions with Variables

A variable is a placeholder for a numeric value(s).

When we evaluate, we are being asked to find a numerical answer. This typically involves substituting numbers into an algebraic expression and then simplifying to get a final answer.

Example:

Evaluate $x + y - 9$ if $x = 15$ and $y = 26$

$x + y - 9 = 15 + 26 - 9$ First, replace x with 15 and y with 26

$= 41 - 9$ Add 15 and 26

$= 32$ Subtract 9 from 41

Practice:

1. $ab^2 + c$

   if $a = 2, b = 4$ and $c = 7$

2. $-y^2 - 3xy$

   if $x = -\frac{5}{6}, y = 12$

3. $x^2 - 2(x - y) - z^3$

   if $x = -3, y = 4$ *and* $z = -1$

4. $-a^2 - 2bc - |c|$

   if $a = -3, b = -5$ *and* $c = 2$

5. $\frac{m^2 - n^2}{(m + n)^2}$

   if $m = 6$ and $n = -8$

6. $\frac{a^2 + b^2}{a - b}$

   if $a = -7$ and b $= -9$

VI. Combining Like Terms

A term is either a single number/variable, or any combination of numbers and variables multiplied together. Terms are separated by addition or subtraction signs. We combine like terms by adding or subtracting as indicated. "Like terms" are terms that have the ***exact same*** combination of variables with the ***exact same*** exponents.

Practice: Simplify each expression by combining like terms.

1. $16x - 11 - 7x + 10$
2. $4x^2 - 7 + 2x - 3x^2 + 8x - 6$
3. $a^2 + 7ab - 10ab + b^2$
4. $5x + 7y - x + 2y + 3x$

VII. Distributive Property

The Distributive Property combines addition and multiplication to represent equivalent expressions. For example, the expressions 3(4 + 2) and 3 · 4 + 3 · 2 are equivalent because they have the same value, 18.

Example a. $\mathbf{2(x-1)}$

$$\begin{aligned} 2(x-1) &= 2[x+(-1)] && \text{Rewrite } x-1 \text{ as } x+(-1) \\ &= 2x+2(-1) && \text{Distributive property} \\ &= 2x+(-2) && \text{Simplify} \\ &= 2x-2 \end{aligned}$$

Example b. $\mathbf{-3(n-5)}$

$$\begin{aligned} -3(n-5) &= -3[n+(-5)] && \text{Rewrite } n-1 \text{ as } n+(-5) \\ &= -3n+(-3)(-5) && \text{Distributive property} \\ &= -3n-15 && \text{Simplify} \end{aligned}$$

Practice: Use the Distributive Property to write each expression as an equivalent algebraic expression.

1. $4(x+3)$
2. $-8(m-4)$
3. $(p+4)5$
4. $x(2+3)$
5. $8(y-2)$
6. $(n+2)3$
7. $(a-6)(-5)$
8. $5(y+8)$

VIII. Translate Word Sentences into Expressions / Equations / Inequalities

When reading through word problems, it is helpful to first **define your variable(s)** by identifying what it is you are trying to solve or find. Then, be on the lookout for key words that can help you identify what operations to use in your equation. The following tables list some words and phrases that you should know:

Addition	Subtraction	Multiplication	Division
plus	minus	times	dividend
sum	difference	product	quotient
more than	less than	multiplied	per
increased by	subtract	each	rate
in all	decreased by	of	ratio
total	less	factors	separate

Expression	Key word(s)	Verbal Phrase
$\mathbf{5 \times 8}$	times	5 times 8
$\mathbf{2+4}$	sum	the sum of 2 and 4
$\mathbf{16 \div 2}$	quotient	the quotient of 16 and 2
$\mathbf{8-6}$	less than	6 less than 8
$\mathbf{2 * 5}$	product	the product of 2 and 5
$\mathbf{5-2}$	less	5 less than 2

Practice: Translate each phrase into an algebraic equation or inequality.

1. Five less than the quotient of a number and three is negative seven. What is the number?
2. Four times the difference between a number and seven is 32. What is the number?
3. One-third of a number is twelve less than the number itself. What is the number?
4. A number that is no more than twenty. Represent as an inequality.

IX. Solving Linear Equations

Solving one and two-step equations is an essential concept in Algebra that involves finding the value of a variable that satisfies an equation.

How to solve these types of equations:

One-step equations: can be solved by performing the same operation on both sides of the equation.

To solve:

1. Identify the operation that is being performed on the variable.
2. Perform the opposite operation on both sides of the equation to isolate the variable.
3. Check the solution by substituting it back into the original equation.

Two-step equations: To solve:

1. Identify the operations that are being performed on the variable.
2. Perform the opposite of the addition or subtraction operation on both sides of the equation.
3. Perform the opposite of the multiplication or division operation on both sides of the equation.
4. Check the solution by substituting it back into the original equation.

Example a.

$x + 3 = 18$	Identify the operation (+3)
$x + 3 \ \mathbf{-3} = 18 \ \mathbf{-3}$	Apply the opposite operation to both sides of the equation ($\mathbf{-3}$)
$\boldsymbol{x = 15}$	← The solution
$\mathbf{15} + 3 = 18$	Substitute your answer back into the original problem to check your answer
$18 = 18$	***Check*** ☑

Example b.

$2x - 10 = -4$	Identify the value to eliminate first (*reverse* PEMDAS) (-10)
$2x - 10 \mathbf{+ 10} = -4 \mathbf{+ 10}$	Apply the opposite operation to both sides of the equation **(+10)**
$2x = 6$	Identify the operation (× 2)
$\frac{2x}{\mathbf{2}} = \frac{6}{\mathbf{2}}$	Apply the opposite operation to both sides of the equation (÷ **2**)
$\boldsymbol{x = 3}$	← The solution
$2(\mathbf{3}) - 10 = -4$	Substitute your answer back into the original problem to check your answer
$6 - 10 = -4$	Simplify
$-4 = -4$	***Check*** ☑

Practice: Solve each one-step equation for x.

1. $x + 26 = 17$
2. $-19x = -209$
3. $-2 + x = 10$
4. $\frac{x}{29} = -7$
5. $15x = -120$
6. $x \div 14 = 2$
7. $2 = x - 14$
8. $x + 7 = -6$

Practice: Solve each two-step equation for x.

9. $11x + 11 = -198$
10. $-8 - 12x = -68$
11. $x \div 22 - 7 = -6$
12. $\frac{x}{3} + 1 = 8$
13. $-11x - 4 = 117$
14. $-5x - 2 = 33$
15. $\frac{x}{1} - 5 = -5$
16. $5x - 4 = 26$

X. Inequalities

Inequalities are used to describe the relationship between expressions that are not equal. They can be presented algebraically, in words, or on a number line.

An inequality can have multiple solutions. When a single inequality is used there can be an infinite number of solutions. In some cases, the number of solutions can be finite. For example, when the unknown value is an integer and between two given values.

Solving inequalities is very similar to solving equations, but if you multiply or divide **by a negative number**, you need to reverse the direction of the inequality symbol. Solutions can be expressed in several different ways, such as **inequality notation**, **interval notation**, or by **graphing on a number line**.

Be able to solve one and two-step inequalities, represent inequalities graphically on a number line, and write inequalities from a number line.

Example:

$$-2x - 8 \geq 5$$

$$-2x - 8 + \mathbf{8} \geq 5 + \mathbf{8} \qquad \text{Add 8 to each side}$$

$$-2x \geq 13 \qquad \text{Divide both sides by } -2$$

$$\frac{-2x}{\mathbf{-2}} \geq \frac{13}{\mathbf{-2}} \qquad \text{Reverse the direction of the inequality}$$

$$x \leq -\frac{13}{2}$$

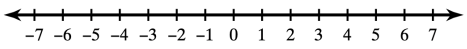
$n > -1$

$n \leq 3$

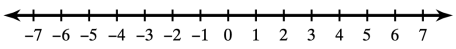
$-1 < n < 3$

Practice: Solve each inequality and represent your answer on the number line.

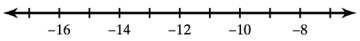
1. $n > 5$



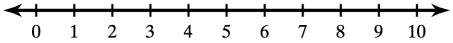
2. $x \leq -5$



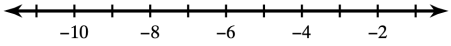
3. $\frac{n}{3} < -5$



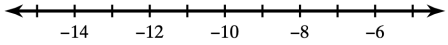
4. $x + 5 < 13$



5. $-8x > 40$

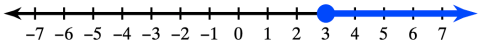


6. $4 + v \leq -6$



For #7-10 write an inequality for each graph:

7.



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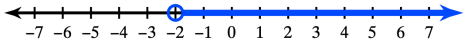

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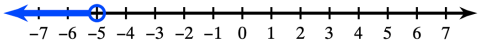

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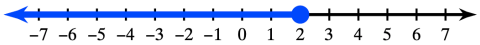


Answers

I. Fact Fluency

1. 20	3. 9	5. 12	7. 8	9. 0
2. 27	4. 56	6. 48	8. 96	10. 7

II. Operations with Integers

1. 14	3. -12	5. -246	7. -87	9. 87
2. 18	4. -84	6. 9	8. -77	10. 7

III. Absolute Value

1. 21	3. 2	5. 3	7. 19
2. 86.5	4. 5	6. 45	8. 13

IV. Order of Operations

1. 15	3. 53	5. 58	7. 8
2. -5	4. 15	6. $-4\frac{2}{3}$	8. -61

V. Evaluating Expressions with Variables

1. 39	3. 24	5. -7
2. -114	4. 9	6. 65

VI. Combining Like Terms

1. $9x - 1$	3. $a^2 - 3ab + b^2$
2. $x^2 + 10x - 13$	4. $7x + 9y$

VII. Distributive Property

1. $4x + 12$	4. $2x + 3x = 5x$	7. $-5a + 30$
2. $-8m + 32$	5. $8y - 16$	8. $5y + 40$
3. $5p + 20$	6. $3n + 6$	

VIII. Translate Word Sentences into Expressions / Equations / Inequalities

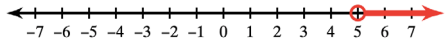
1. $\frac{x}{3} - 5 = -7;\ x = -6$	3. $\frac{1}{3}x = x - 12;\ x = 24$
2. $4(x - 7) = 32;\ x = 15$	4. $x \leq 2$

IX. Solving Linear Equations

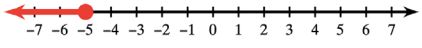
1. -9
2. 11
3. 12
4. -203
5. -8
6. 28
7. 16
8. -13
9. -19
10. 5
11. 22
12. 21
13. -11
14. -7
15. 0
16. 6

X. Inequalities

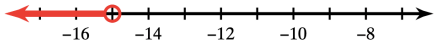
1. $n > 5$



2. $n \leq -5$

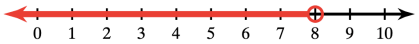


3. $\frac{n}{3} < -5$



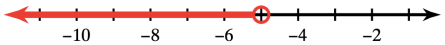
$n < -15$

4. $x + 5 < 13$



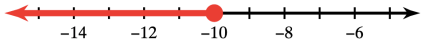
$x < 8$

5. $-8x > 40$

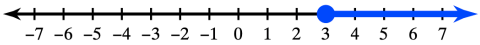


$x < -5$

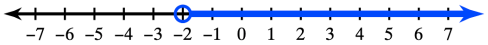
6. $4 + v \leq -6$



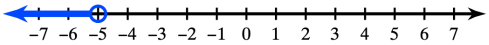
$v \leq -10$

7. 

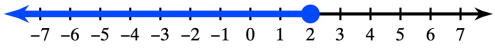
$x \geq 3$

8. 

$n > -2$

9. 

$x < -5$

10. 

$n \leq 2$